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FINAL STATE INTERACTION CORRECTIONS TO INELASTIC NUCLEON-NUCLEON SCATTERING

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Abstract: The final state interaction corrections to the peripheral model calculation of Drell and Hiida of the process $n+n \rightarrow n+n+\pi$ are calculated. Methods of calculation and comparison with experiment are discussed.

1. Introduction

The differential scattering cross section of inelastically scattered nucleons on nucleons has been computed in the peripheral model approximation by Drell and Hiida [1]). Their calculation gives a mechanism for the explanation of the peak obtained in the experiments of Cocconi *et al.*[2]). The dominant contribution to the inelastic scattering is assumed to come from the process $n+n \rightarrow n+n+\pi$ via the peripheral graph of fig. 1, where the vertex A is taken to be a diffraction scattering amplitude. The above

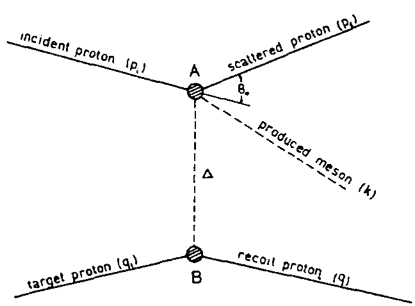

Fig. 1. Peripheral graph for the process $n+n \rightarrow n+n+\pi$.

calculation has two ambiguities: (1) the shape of the diffraction peak in π-n scattering is not known to a very high precision; (2) the form factor of vertex B and pion propagator (fig. 1) is also somewhat arbitrary. The first point will be discussed further. By choosing the diffraction peak and cut off on the momentum transfer integration, i.e. form factor at vertex B, one may obtain a good overall fit with experiment without reproducing any "fine structure" as shown in ref. [2]).

The purpose of this paper is to study the effects of final state interaction between the

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produced pion and the recoil nucleon. The procedure consists of expanding the matrix element of fig. 1 in partial wave amplitudes of the system formed by the recoil nucleon and the produced pion and putting in the relevant final state corrections. The method is analogous to the one used by Jacob, Omnès and Mahoux [3]).

A similar calculation was made by Islam [4]), who considered only the contributions due to S and P waves in the π-n system. This calculation suffers from the lack of knowledge of the scattering phase shifts in these waves at higher energies (above 500 MeV). By assuming a constant average complex phase shift Islam obtained to a large extent a constant multiplicative correction, which in his case amounts to approximately doubling the uncorrected cross section.

If the above calculation is repeated with a more precise phase shift input, and the above qualitative result duplicated, this will cast doubt on the entire peripheral model approach to strong interactions. In the present calculation it is assumed that the major contribution to π-n scattering in the laboratory energy region of 600 MeV to 1100 MeV, in the channel of isobaric spin $\frac{1}{2}$ comes from the D and F wave resonances at approximately 600 and 900 MeV [5]). The purpose of the calculation is twofold:

(i) To reproduce the camel's bump fine structure of ref. [2]), which may correspond to the above mentioned resonances.

(ii) The peak of the cross section curve obtained by Drell and Hiida is always situated to the left of the experimental one. It is hoped that final state corrections may shift the theoretical peak somewhat to the right.

The calculations presented in this article are for $\theta_0 = 40$ mrad, incident energy 16 GeV and 25 GeV, and for $\theta_0 = 56$ mrad, incident energy 16 GeV. In sect. 2 we discuss the procedure for this calculation, and in sect. 3 we state the results and conclusions.

2. Calculation

The differential cross section we are interested in is $\mathrm{d}^2\sigma/\mathrm{d}E_\mathrm{f}\,\mathrm{d}\Omega_\mathrm{f}$ at fixed incident energy, fixed angle θ_0, and as a function of the scattered proton momentum. We choose the following five Lorentz invariant quantities to characterize the process:

$t = (p_\mathrm{i}-p_\mathrm{f})^2$: momentum transfer squared at vertex A;

$W = (p_\mathrm{i}+q_\mathrm{i})^2$: centre of mass energy squared of the whole system;

$V = (k+q)^2$: centre of mass energy squared of the recoil nucleon and produced pion;

$\Delta^2 = (q_\mathrm{i}-q)^2$: momentum squared of virtual pion;

$S_1 = (p_\mathrm{f}+k)^2$: centre of mass energy squared of virtual pion and incident proton.

Our metric is $g_{11} = g_{22} = g_{33} = -g_{00} = 1$, and our units are such that $\hbar = c =$ mass of nucleon $= 1$.

The matrix element M of fig. 1 is

$$\frac{1}{(2\pi)^{\frac{9}{2}}}\,\bar{u}(q)Bu(q_i)\,\frac{1}{\Delta^2+\mu^2}\,\bar{u}(p_f)Au(p_i)\,\frac{\mathrm{d}^3p_f}{\sqrt{2E_f}}\,\frac{\mathrm{d}^3q}{\sqrt{2E_q}}\,\frac{\mathrm{d}^3k}{\sqrt{2E_k}}\,.$$

Following ref. [1]) we take

$$B = ia\sqrt{4\pi}\,\frac{f}{\mu}\,\gamma_5 F(\Delta^2),$$

where a is 1 for π^0 and $\sqrt{2}$ for $\pi^\pm$, $f = 0.08$, and $F(\Delta^2)$ is the form factor for vertex B. For A we take a diffraction scattering matrix element of the form

$$A = i\sigma_{\mathrm{total},\,\pi\text{-n}}(S_1)(S_1+1)\sqrt{g(t)/(4+t)},$$

where $g(t)$ gives the shape of the diffraction peak. Although some more recent theories [6]) suggest that the diffraction scattering matrix element should have the form $\beta(t)S_1^{\alpha(t)}$, our interest is to find the corrections to Drell and Hiida's result [1]), and we therefore take the same parameters as these authors.

The next step consists of going to the centre of mass of $k+q$, effecting a two-component Pauli spinor projection on vertex B, and expanding the direct and spin flip amplitudes as power series in $Y_l^m(\vartheta, \varphi)$, where ϑ is the angle between q and q_i, and φ is the azimuthal angle of q with respect to the plane of p_i, q_i, p_f. The vertex A does not enter into the partial wave projection. The partial amplitudes are recombined into a form where the total angular momentum j and its projection m_j are good quantum numbers, and the Omnès-Muskhelishvili factor

$$\exp\left\{-\frac{\mathrm{P}}{\pi}\int_{-1-\mu}^{-\infty}\frac{\delta(\nu')}{\nu'-\nu}\,\mathrm{d}\nu'\right\}$$

is introduced. We are interested only in D and F waves in the $T = \frac{1}{2}$ channel and we assume their phase shifts to be dominated by the D resonance at $\nu_R = 600$ MeV, $\Gamma = 100$ MeV and the F wave resonance at $\nu_R = 900$ MeV, $\Gamma = 100$ MeV. For $\delta(\nu)$ we take the Breit-Wigner form

$$\delta(\nu) = \frac{1}{2i}\log\frac{\nu-\nu_R+\frac{1}{2}i\Gamma}{\nu-\nu_R-\frac{1}{2}i\Gamma}\,.$$

All the relevant computations were performed on the CERN 7090 computer.

3. Conclusions

The results of the calculation are presented in figs. 2, 3 and 4. Instead of plotting the cross section as a function of energy of the scattered proton, we plot it as a function of $\varepsilon = E_i - E_f + \frac{1}{2}t$ (see ref. [1])). In the case of $\theta_0 = 56$ mrad we find some of the fine structure present in the experimental data. The shift of the peak is also present but is not sufficient to make the two peaks coincide.

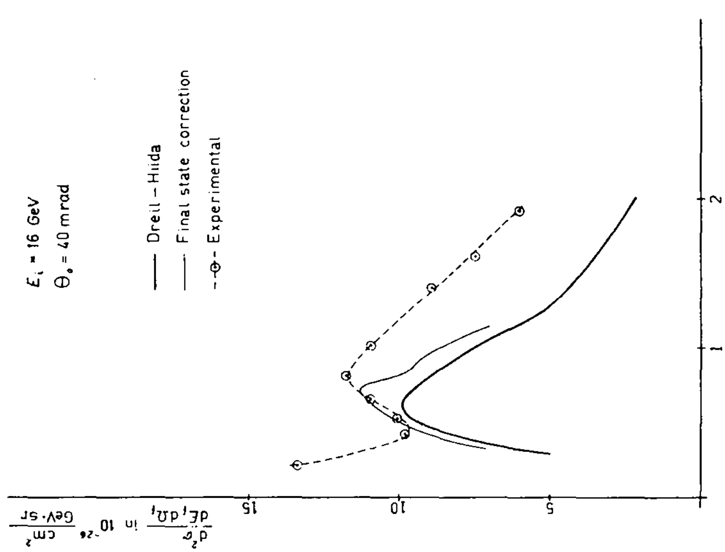

Fig. 2. Scattered proton momentum spectrum.

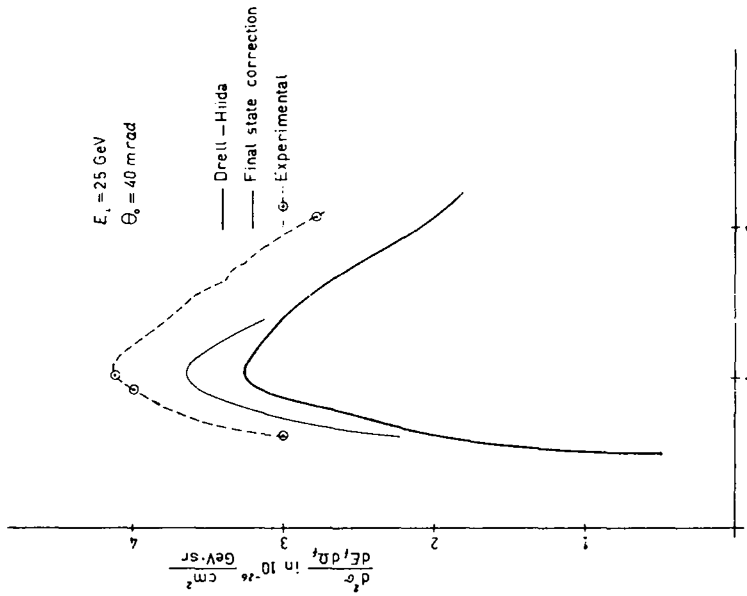

Fig. 3. Scattered proton momentum spectrum.

The amount of D and F waves is approximately 10 and 6 percent at 40 mrad and 25 and 13 percent at 56 mrad. We expect that as we go to larger angles we pick up more of the higher waves, as at small angles we have predominantly P and some S wave due to the γ_5 interaction at vertex B, and there is no change in partial wave distribution in a small angle diffractive scattering at vertex A. Also, as at vertex B, the pion and proton emerge in a $T = \frac{1}{2}$ state, there is no isobaric flip at vertex B, and the pion and recoil nucleon remain in a $T = \frac{1}{2}$ state.

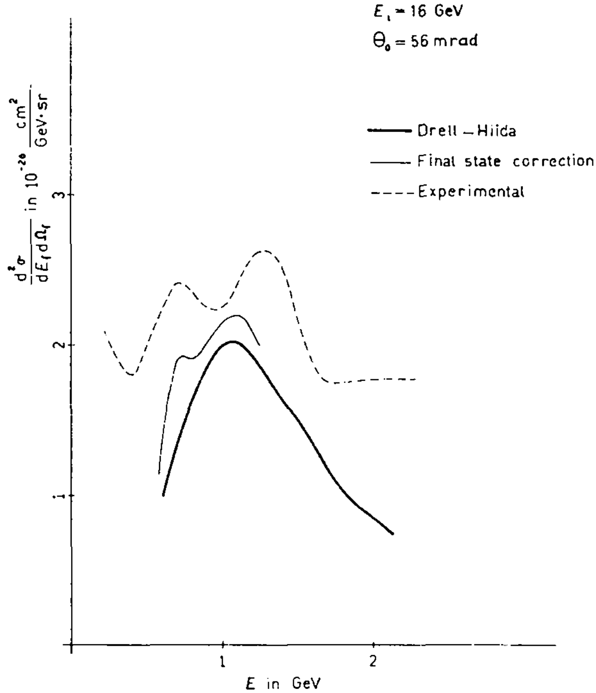

Fig. 4. Scattered proton momentum spectrum.

This calculation will be extended to other angles, and several diffraction and momentum transfer parameters will be used.

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References

1) S. D. Drell and K. Hiida, Phys. Rev. Lett. 7 (1961) 199
2) G. Cocconi, A. N. Diddens, E. Lillethun and A. M. Wetherell, Phys. Rev. Lett. 6 (1961) 231
3) M. Jacob, G. Mahoux and R. Omnès, Nuovo Cim. 23 (1962) 838
4) Islam, preprint, Imperial College London (1962)
5) W. Layson, Interpretation of pion-nucleon scattering results, CERN preprint (1962)
6) C. Lovelace, quoted by S. D. Drell in Proceedings of the International Conference on Elementary Particles, 1961 (Commissariat à l'Énergie Atomique, Saclay, Gif-sur-Yvette, 1962), Vol. II, p. 128